NUMERICAL EVALUATION OF THE BEHAVIOR OF A PLATE ON IMPACT WITH A RIGID PROJECTILE USING ELEMENT-FREE GALERKIN METHOD

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DOI: 10.19062/2247-3173.2022.23.1

Abstract: *The impact of the projectile-plate is a complex phenomenon that is analyzed through analytical methods, based on simplifying hypotheses. In addition to the use of empirical laws, these aspects of the projectile-plate interaction and the effects on the structure are studied using numerical methods. This article presents the numerical evaluation of the behavior of a monolithic plate on impact with a rigid projectile using element-free Galerkin method and is shown the evolution of the impact with its effects (deformation with perforation of the plate). Also, an analyze of the variation of the total energy of the plate, kinetic energy of the bullet and bullet velocity over time are presented.*

Keywords: *impact, plate, projectile, element free galerkin method*

1. INTRODUCTION

The impact of the projectile-plate is a complex phenomenon that is analyzed through analytical methods, based on simplifying hypotheses. In addition to the use of empirical laws, these aspects of the projectile-plate interaction and the effects on the structure are studied using numerical methods.

Significant research has been conducted on the behavior of composite materials on impact. However, research on ballistic impact is still in an incipient phase.

"The Element-Free Galerkin (EFG) method is a very promising method for the treatment of partial differential equations. Because of the absence of element connectivity, nodal points can be added easily to the part of the domain where the solution is (expected to be) steep. This makes the EFG-method more flexible than the Finite Element (FE) method. The method looks very promising for computations in fracture mechanics, since nodal points can be arranged around crack tips in order to obtain accurate stress intensity factors" [1].

The element-free Galerkin Method is based on a series of equations of the theory of elasticity, used under special conditions of numerical approximation, by the method Mooving Least Squares or MLS [2].

A mesh free method establishes system algebraic equations for the entire problem area without using a preset mesh for domain discretization.[3]

2. FUNDAMENTALS OF THE EFG METHOD

The moving least-squares approximation of a function representing a field variable is used in the Element-free Galerkin technique [3]. The approximated value of $u(x)$ will be denoted by $u^h(x)$ represented by the expression:

$$u^h(x) = \sum_{i=1}^{n} H_i(x) \cdot b_i(x) \tag{1}$$

In a matrix form, relation (1) is written:

$$u^h(x) = \mathrm{H}^{\mathrm{T}}(\mathrm{x})\mathrm{b}(\mathrm{x}) \tag{2}$$

where n is the order of the completeness in this approximation, the monomial $H_i(x)$ are basis functions and $b_i(x)$ are the coefficients of the approximation function.

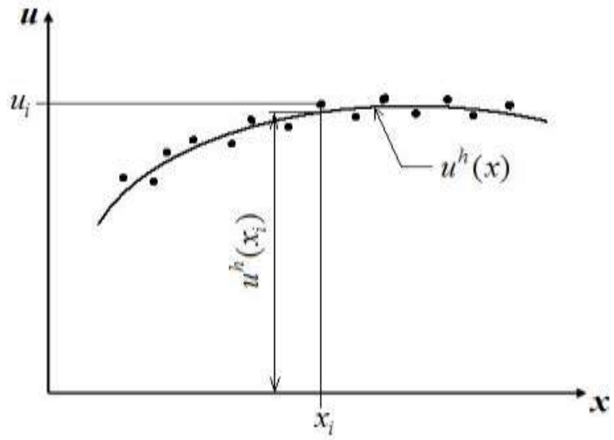

FIG. 1 Nodal parameters u_i and approximate function $u^h(x_i)$

As seen in Fig. 1, there is a difference between the nodal parameter and its approximated value for a node i in the moving least-squares approximation. The coefficients $b_i(x)$ for a point **x** depend on the sampling points $\mathbf{x_I}$ which are selected by a weighting function $w_a(\mathbf{x}\text{-}\mathbf{x_I})$. A weighting function is defined on a compact support defined by a sub-domain. Each sub-domain Ω_I is associated with a node I. Often a such sub-domain is a circle or a ball (3D space), like in the Fig. 2.

The moving least-squares technique is based on minimizing the weighted L_2-Norm (J) defined by the relation (3) or (4); NP is the number of nodes (points) within the support domain where $w_a(\mathbf{x}\text{-}\mathbf{x_I}) \neq 0$.

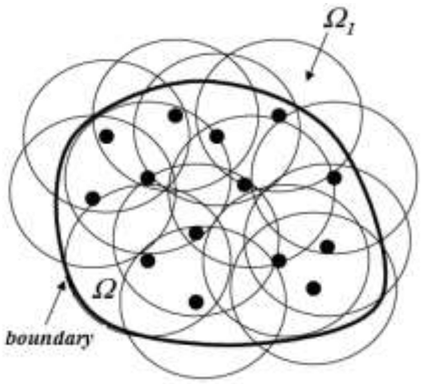

FIG. 2 A mesh-free discretization

$$J = \sum_{I=1}^{NP} W_a(x)(x - x_I)\left[u^h(x) - u_i(x_I)\right]^2 \tag{3}$$

$$J = (\mathbf{Hb} - \mathbf{u})^T W_a(\mathbf{x})(\mathbf{Hb} - \mathbf{u}) \tag{4}$$

In the relations (3) and (4) the following notations have been used:

$$\mathbf{u}^T = (u_1, u_2, ..., u_{NP}) \tag{5}$$

$$\mathbf{H} = \begin{bmatrix} \{H(x_1)\}^T \\ ... \\ \{H(x_{NP})\}^T \end{bmatrix} \tag{6}$$

$$\{H(x_i)\}^T = \{H_1(x_i), ..., H_n(x_i)\} \tag{7}$$

$$W_a = \mathrm{diag}[w_a(x - x_1), ..., w_a(x - x_{NP})] \tag{8}$$

The coefficients **b** result from equation:

$$\frac{\partial J}{\partial b} = M^{[n]}(x)b(x) - B(x)u = 0 \tag{9}$$

where,

$$M^{[n]}(x) = H^T W_a(x) H \tag{10}$$

$$B(x) = H^T W_a(x) \tag{11}$$

resulting:

$$b(x) = M^{[n]^{-1}}(x)B(x)u \tag{12}$$

Using the solution of the equations (1), (10), (11) and (12) the EFG approximation is obtained:

$$u^h(x) = \sum_{I=1}^{NP} \Psi_I(x) u_I \tag{13}$$

$\Psi_I(x)$ are shape functions having the expressions:

$$\Psi_I(x) = H^T(x) M^{[n]^{-1}}(x) B(x) \tag{14}$$

The weight function can theoretically be chosen at random as long as certain conditions are met. The most important synthetic conditions are: to be greater than zero within the support domain; to be zero outside the support domain; to be monotonically decreasing from the point of interest; and sufficient smoothness, particularly on the boundary. The most used weight functions are: the cubic and the quartic spline functions.

3. MATERIALS AND METHODS

The purpose of this paper is to evaluate the performance of an aluminum plate on impact with a 7.62 mm rigid projectile using element-free Galerkin method. A normal impact was considered, with an impact velocity of 500 m/s and the analyses time of $9*10^{-5}$ seconds.

Numerical simulations were carried out using the LS-DYNA software [4].

For the theoretical study, the aluminum homogeneous and isotropic plate, presented in Fig.3, has the following characteristics:

- Density: $\rho = 2710$ [kg/m^3]
- Young's modulus: $E = 0.690*10^{11}$ [Pa]
- Poisson's ratio: $\upsilon = 0.33$
- Yield stress: $\sigma_c = 315e6$ [Pa]
- Dimensions: 0.1 m x 0.1 m x 0.005 m
- Volume = 2.588e-5 [m^3]
- Node number = 61206
- Element number (SOLID164) = 50000
- Average element finit dimension = 0.001 [m]

The material model used for the plate was plastic kinematic hardening and a rigid material was considered for the bullet.

The plate was simulated by element-free Galerkin method and the nodes belonging to the four sides have all degrees of freedom blocked (DOF=0).

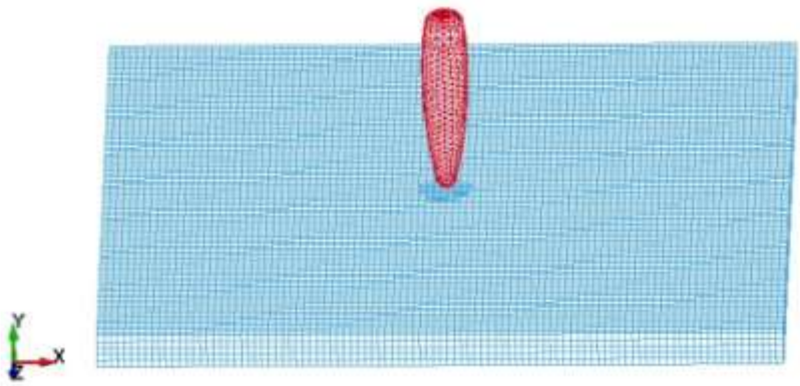

FIG. 3 Element-free Galerkin model

The interest was focused on the plate, that's why it was considered a rigid material for the bullet. The using of these assumptions covers the calculation results and save computer time.

The characteristics of the bullet, presented in Fig. 4, are the following:

FIG. 4 Model of the bullet

- Caliber = 7.62 [mm]
- Density: ρ = 7850 [kg/m^3]
- Impact velocity = 500 [m/s]
- Volume = 6,8587e-7 [m^3]
- Mass = 0.00538 [kg]
- Node number = 6046
- Element number(SOLID168) = 3860
- Average element finit dimension = 0.001 [m]

4. NUMERICAL SIMULATION

In the Fig. 5 it is shown the evolution of the impact, with its effects (deformation with perforation of the plate), by presenting the deformed state during the analysis of 60 microseconds

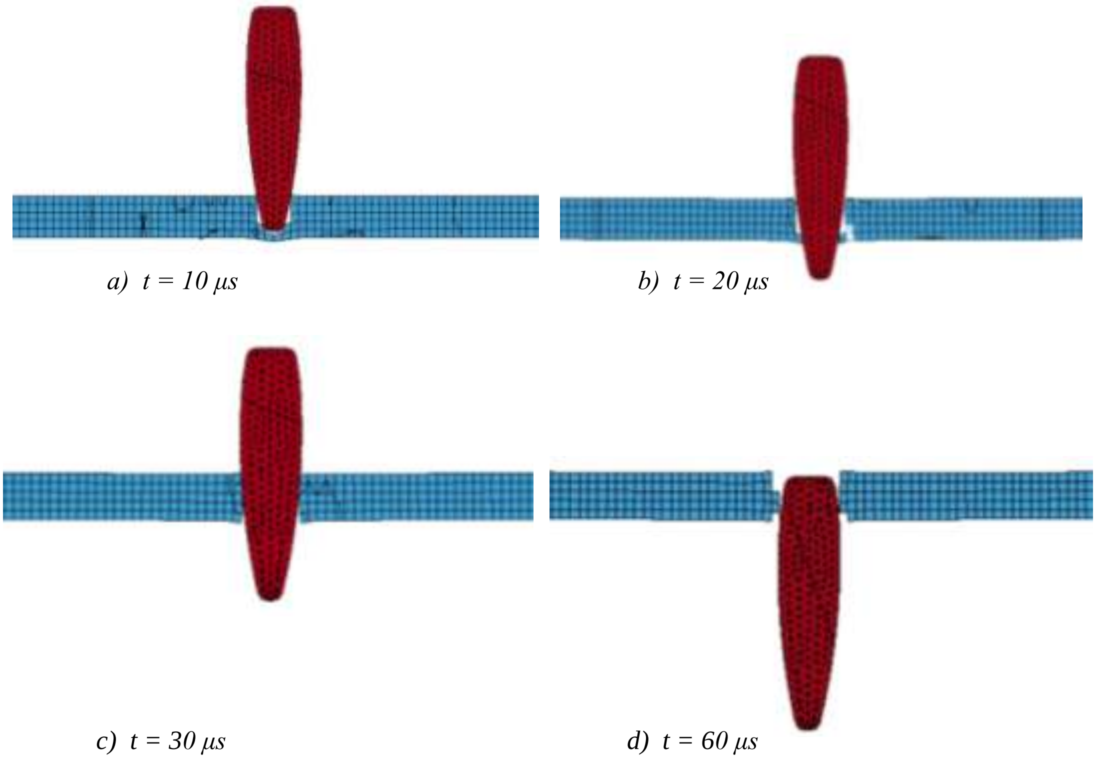

FIG. 5 Time evolution of the impact

The time evolution of the plate total energy is presented in Fig. 6. It is observed that the total energy absorbed by the plate during the impact is reaching a maximum value of 10.5 Nm.

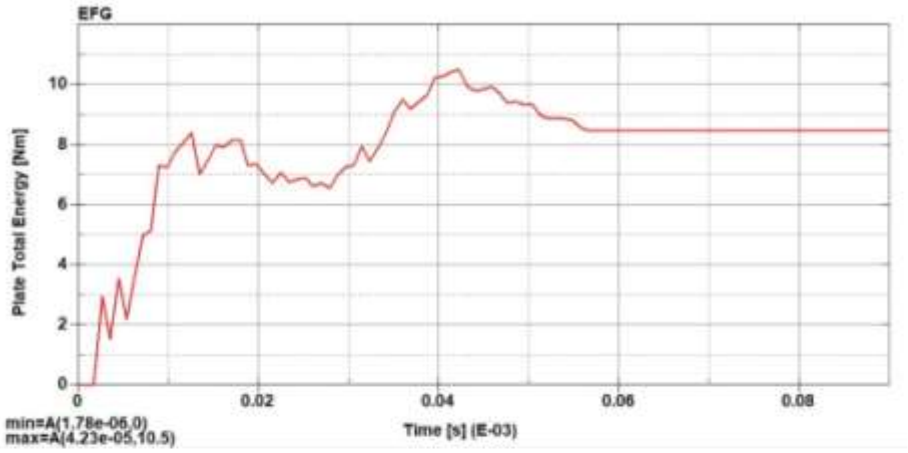

FIG. 6 Time evolution of the plate total energy

From the graphical representation of the bullet kinetic energy variation, presented in Fig. 7, results a variation between the limits of 594-673 Nm, meaning that there is a falling of the kinetic energy of the bullet by 11% and represents the remaining ability of the bullet to pierce or penetrate a plate similar in material and thickness.

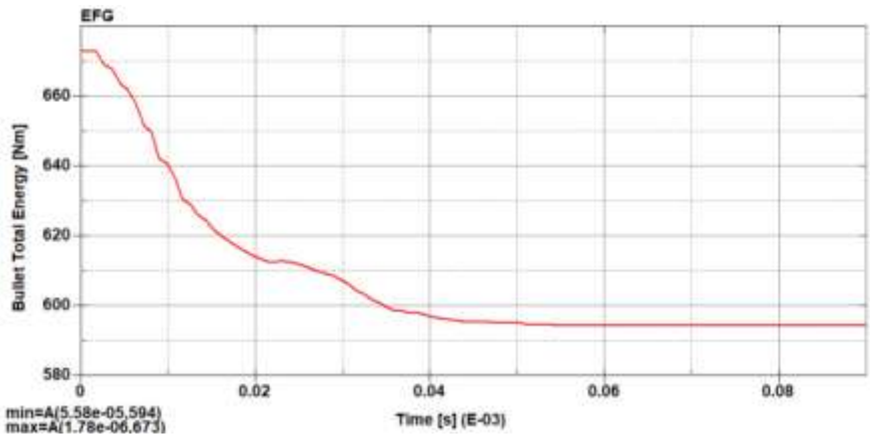

FIG. 7 Time evolution of the bullet total energy

Analyzing the allure of the curve in Fig. 8, can be observe a constant level at the beginning of the diagram that represents the period elapsed to cover the initial bullet-plate distance, then begins the process of penetration and perforation of the plate, the speed of the bullet decreasing from the initial value of 500 m/s to at the minimum value of 470 m/s.

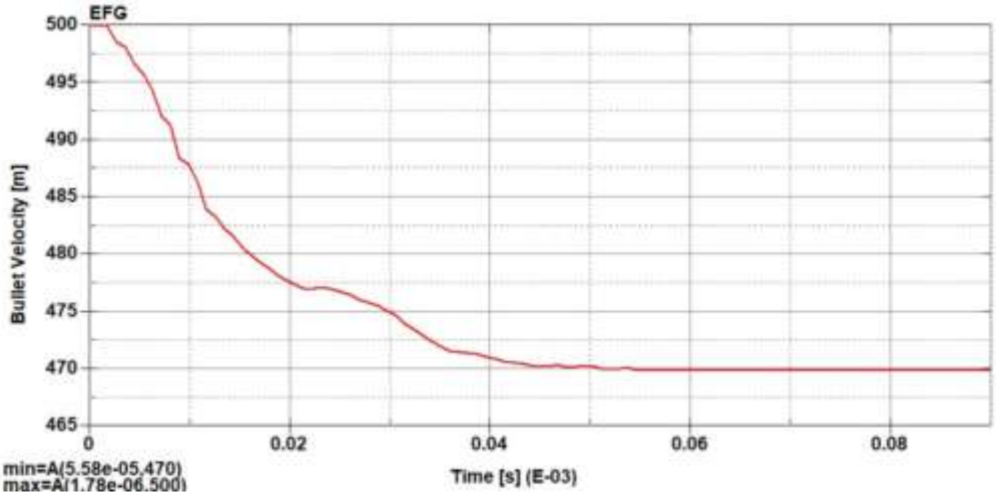

FIG. 8 Time evolution of the bullet velocity

5. RESULTS AND DISCUSSIONS

The results obtained by the element-free Galerkin method were introduced in Table 1, in order to be compared with the results obtained in the numerical simulation of the same impact, analyzed with the Finite Element method and the Smoothed-particle hydrodynamics method.

It can be seen that the values obtained are close and the errors are relatively small, below 10%, which is a very good match of the values obtained, implicitly a proper analysis.

Table 1. Comparison between three numerical methods

	EFG	MEF	SPH	Error EFG/MEF	Error SPH/MEF
Plate total energy [Nm]	10.5	9.89	10.9	6.17%	9.27%
Bullet total energy - max [Nm]	673	673	673	0.00%	0.00%
Bullet total energy - min [Nm]	594	604	543	-1.66%	-8.59%
Bullet velocity - min [m/s]	470	474	449	-0.84%	-4.47%
Bullet residual velocity [m/s]	470	476	450	-1.26%	-4.26%
processing time [s]	233	76	20		

6. CONCLUSIONS

A numerical investigation of the ballistic performance of aluminum plate on impact with 7.62-mm projectile was conducted, using the element-free Galerkin method, for the velocity of 500 m/s. A corresponding experimental study would be expensive and difficult.

The results obtained by elements-free Galerkin method were compared with the results obtained obtained in the numerical simulation of the same impact, analyzed with the Finite Element method and the Smoothed-Particle Hydrodynamics method and the errors are slightly lower, below 10%, this representing a very good concordance of the values obtained, implicitly an appropriate analysis.

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